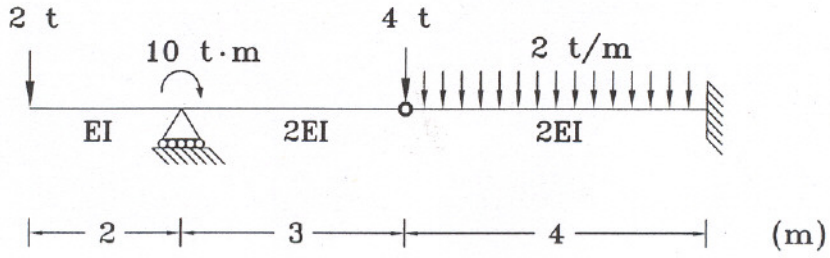
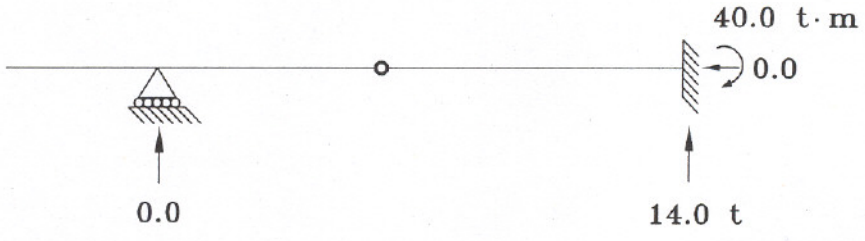
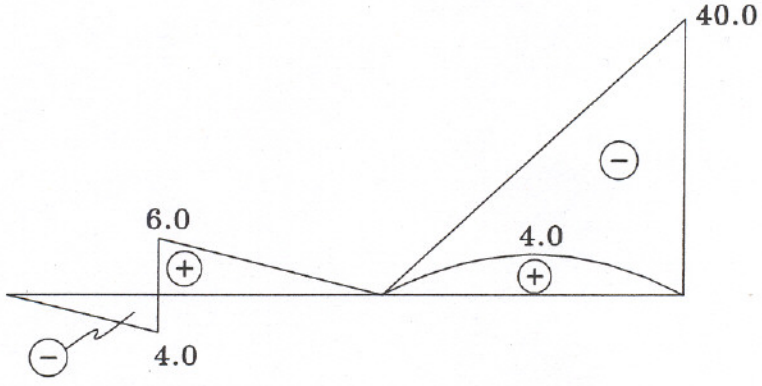
Problema 2.1

Obtener la energía de deformación por efecto de flexión de la estructura que se muestra a continuación.



Resultados:





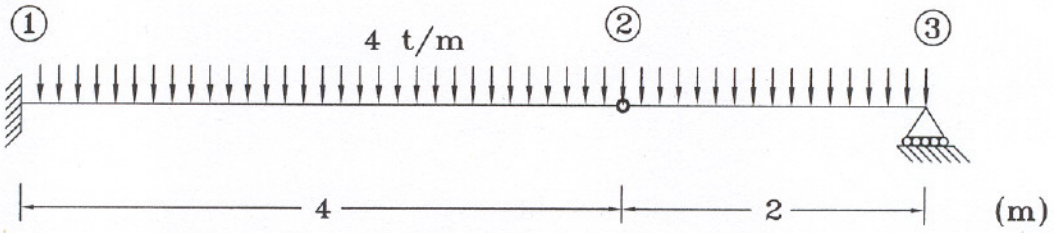
D.M.F.
(ton−m)

$$U = \frac{1}{2}\int \frac{M^2}{EI}\,dx$$

$$U = \frac{449.533}{EI}$$

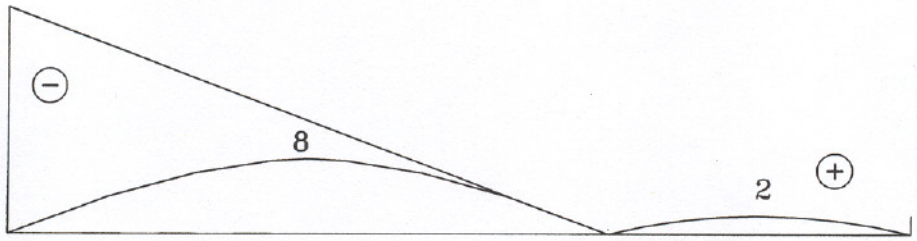
Problema 2.2

De la siguiente estructura, obtener el desplazamiento vertical del punto ②, aplicando el método de trabajo virtual. Considere solo flexión.

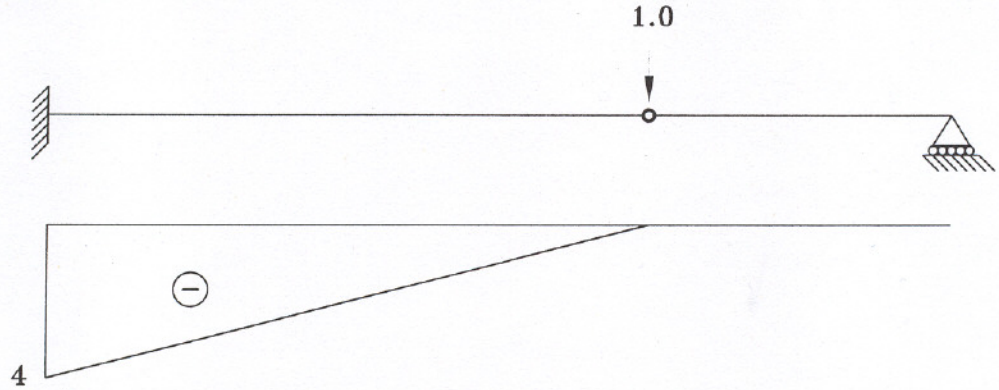


Resultados:

48



D.M.F.
(ton-m)

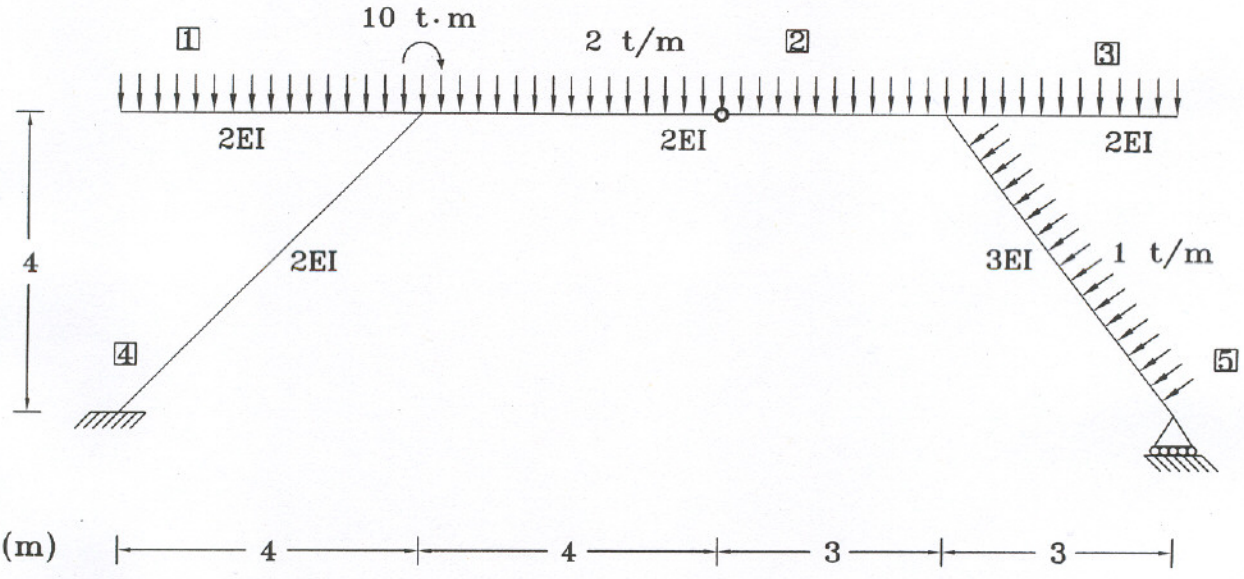


m
(ton-m)

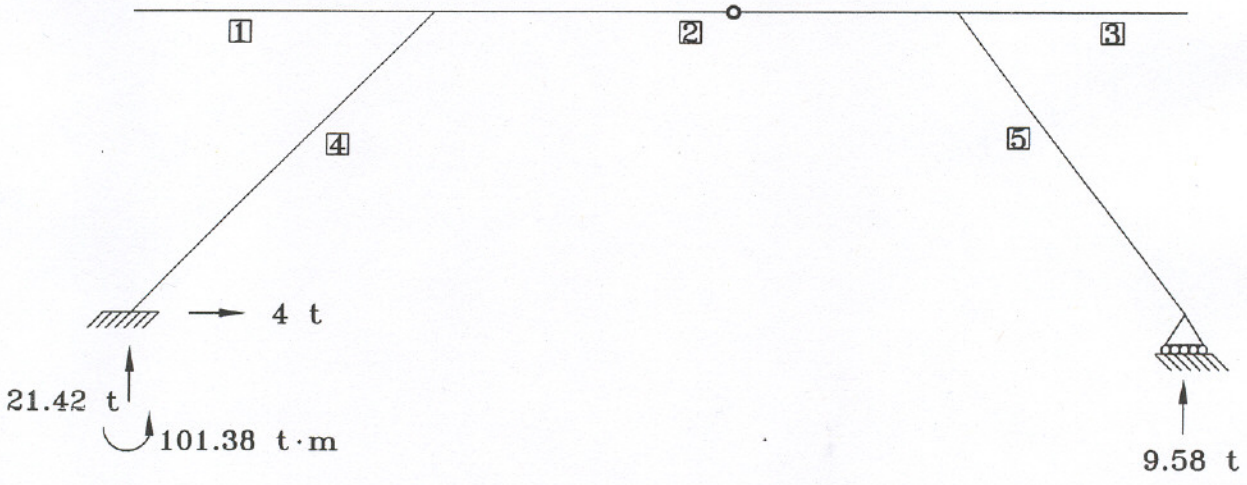
$$d_{2v} = \int \frac{Mm}{EI}\, ds = \frac{213.33}{EI} \quad (\downarrow)$$

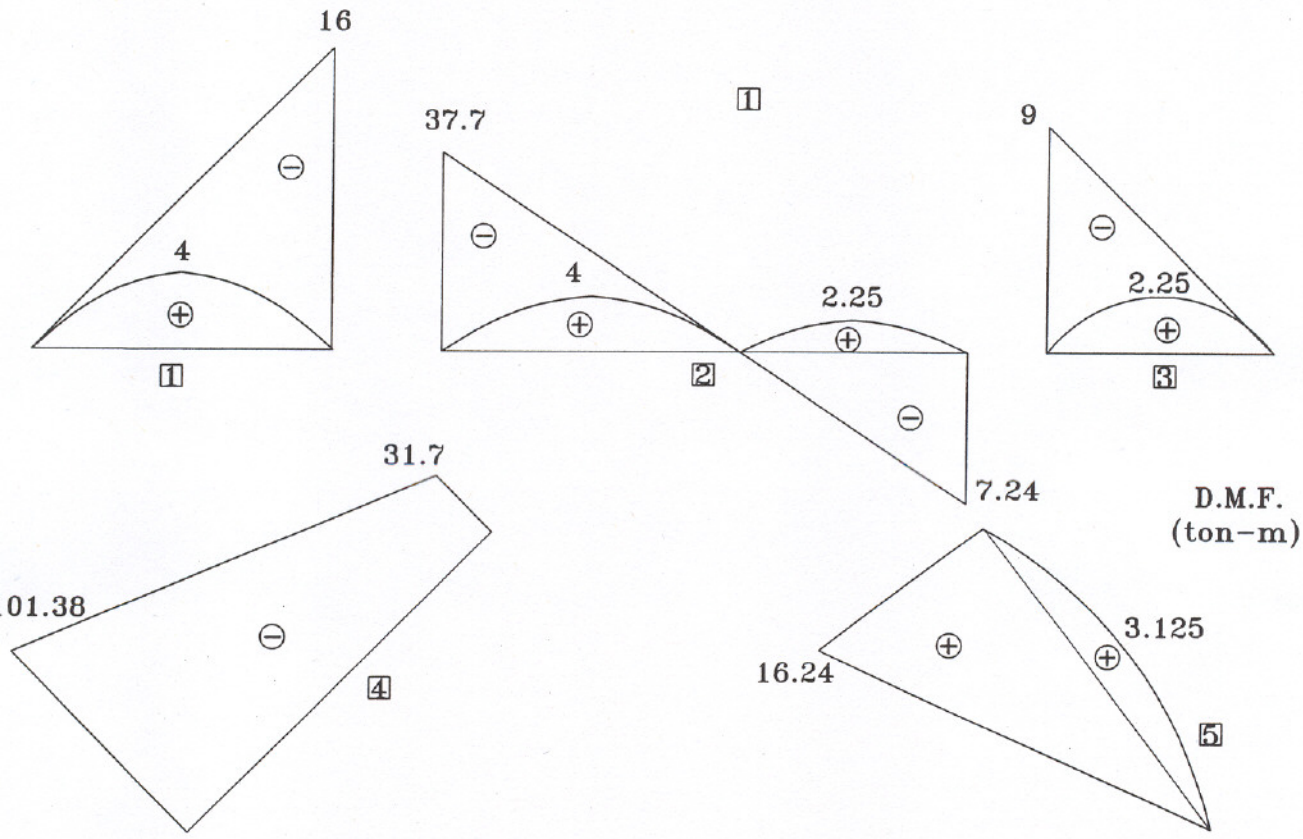
Problema 2.3

Obtener la energía de deformación por efecto de flexión de la estructura que se muestra a continuación.



Resultados:





$$Uf = \frac{1}{2} \int \frac{M^2}{EI}\, dx$$

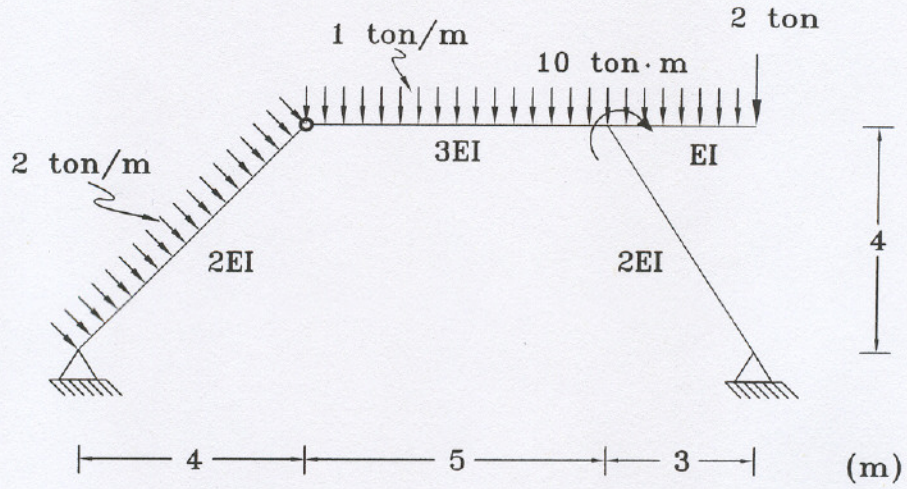
$$Uf_1 = \frac{51.20}{EI} \qquad Uf_4 = \frac{12.15}{EI}$$

$$Uf_2 = \frac{6825.45}{EI} \qquad Uf_5 = \frac{105.29}{EI}$$

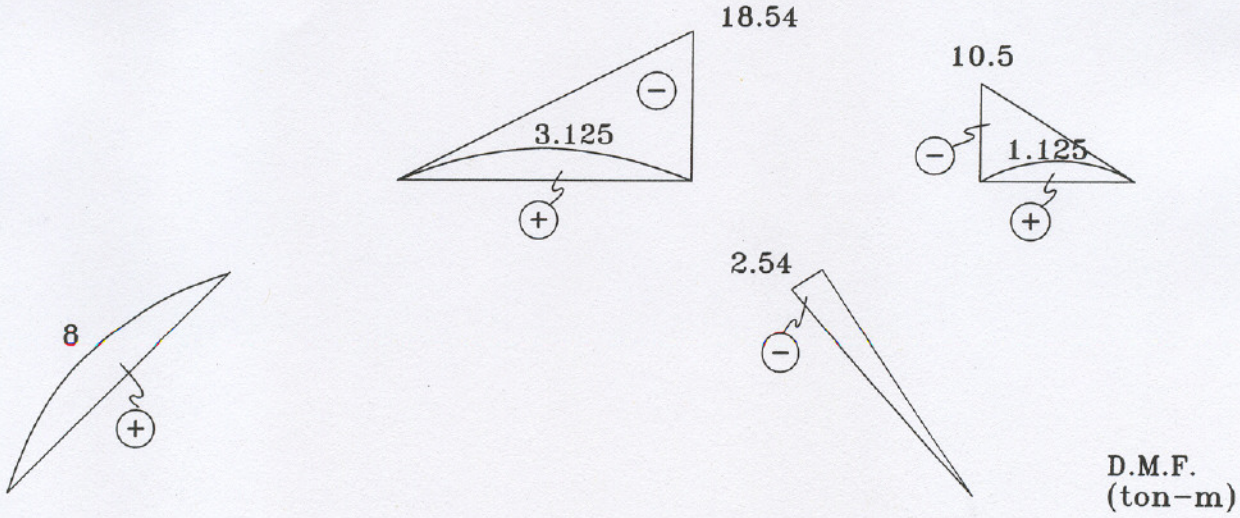
$$Uf_3 = \frac{405.03}{EI} \qquad Uf = \sum_{i=1}^{n} Uf_i = \frac{7399.12}{EI}$$

Problema 2.4

Obtener la energía de deformación por efecto de flexión de la estructura que se muestra a continuación.



Resultados:

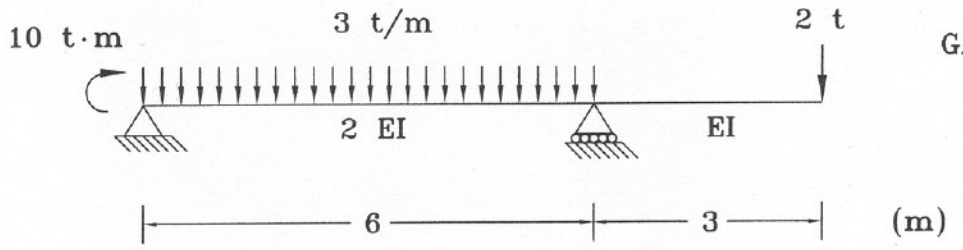


$$Uf = \frac{1}{2}\int \frac{M^2}{EI}\,dx$$

$$Uf = \frac{136.59}{EI}$$

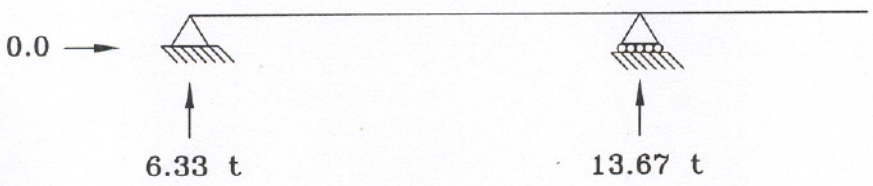
Problema 2.5

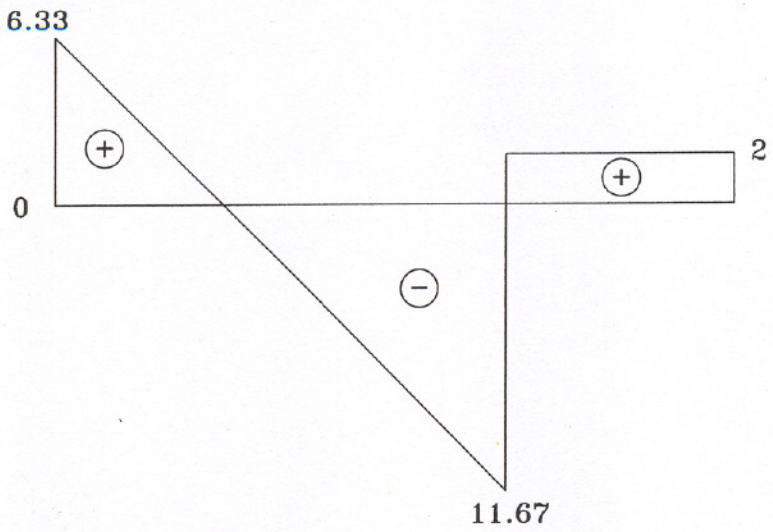
Obtener la energía de deformación por efecto de cortante de la estructura que se muestra a continuación.



GAc =

Resultado:





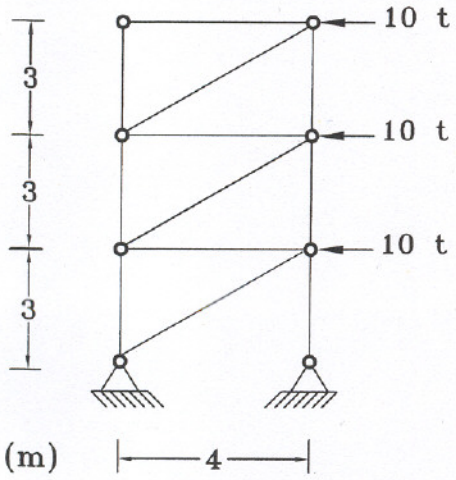
D.F.C. (ton)

$$U_c = \frac{1}{2} \int \frac{V^2}{GAc}\, dx$$

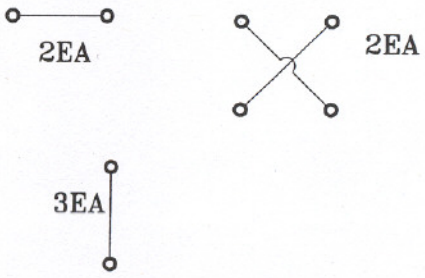
$$U_c = \frac{101.29}{GAc}$$

Problema 2.6

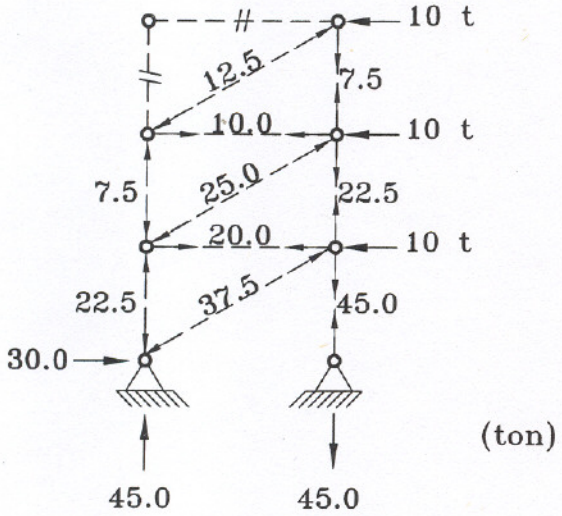
Obtener la energía de deformación de la siguiente estructura.



Propiedades de barras:



Resultados:



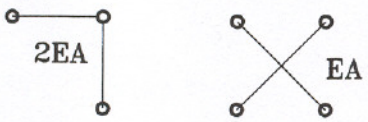
$$Ua = \frac{1}{2}\int \frac{N^2}{EA}\,dx$$

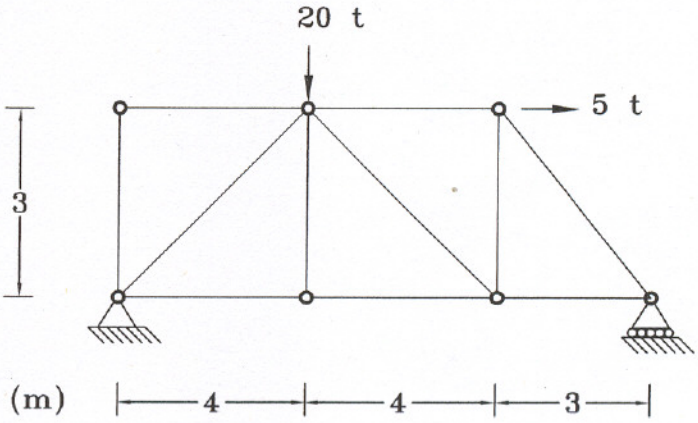
$$Ua = \frac{4809.38}{EA}$$

Problema 2.7

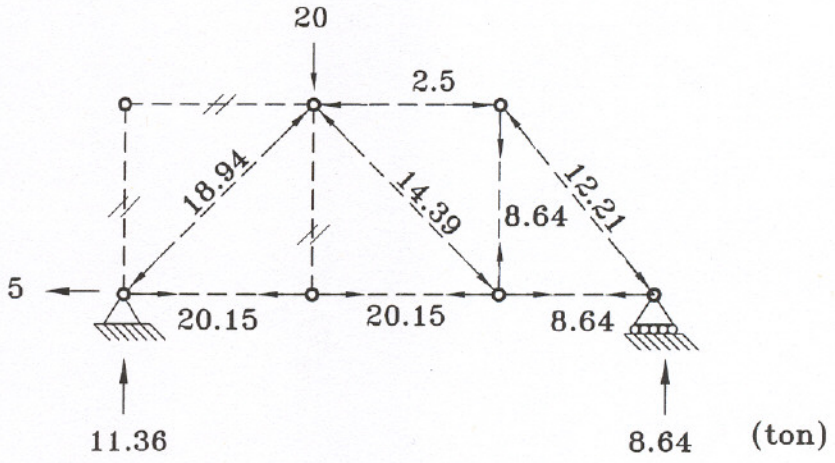
Obtener la energía de deformación de la siguiente estructura.

Propiedades de barras:





Resultado:

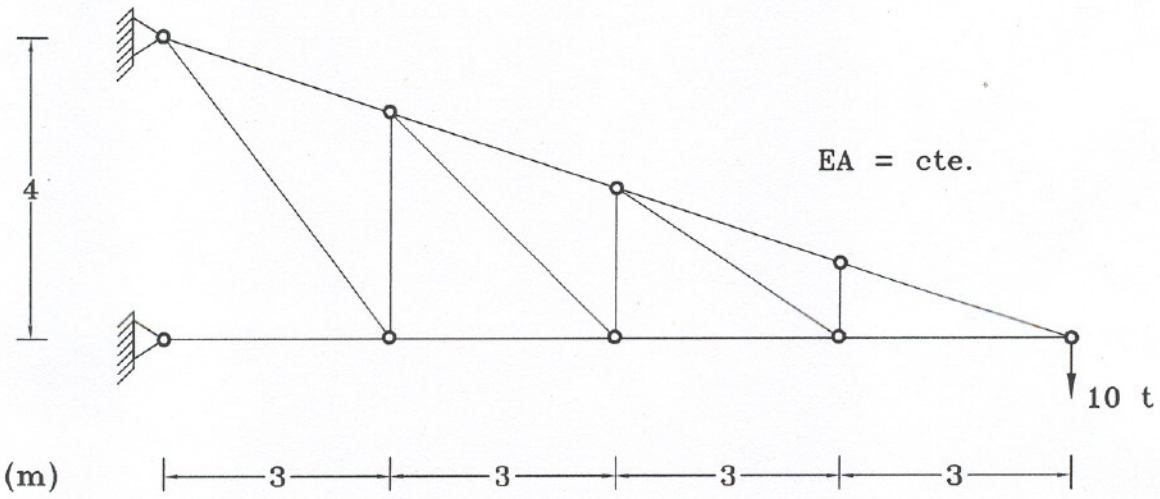


$$Ua = \frac{1}{2}\int \frac{N^2}{EA}\,dx$$

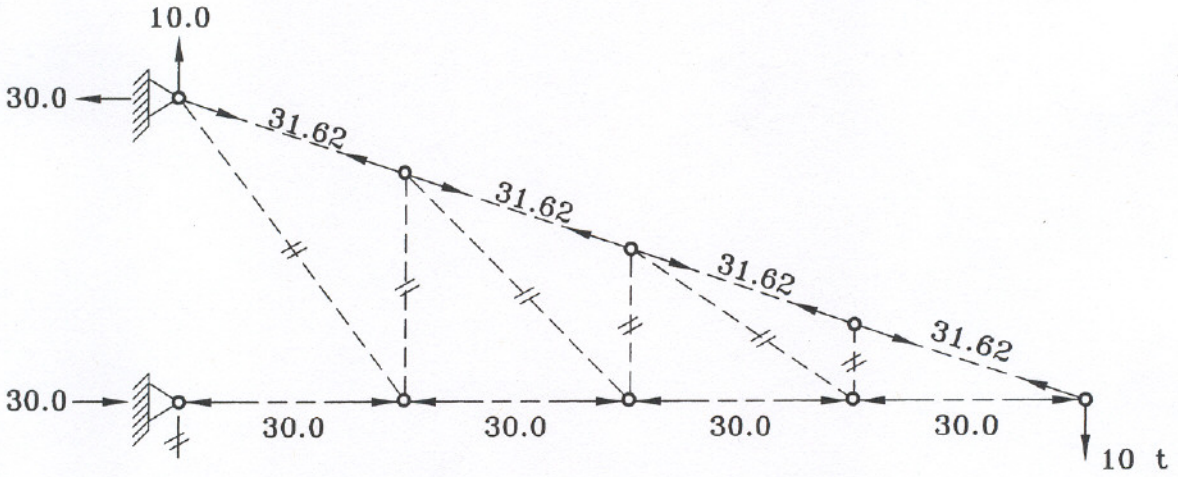
$$Ua = \frac{21668.01}{EA}$$

Problema 2.8

Obtener la energía de deformación de la siguiente estructura.



Resultados:

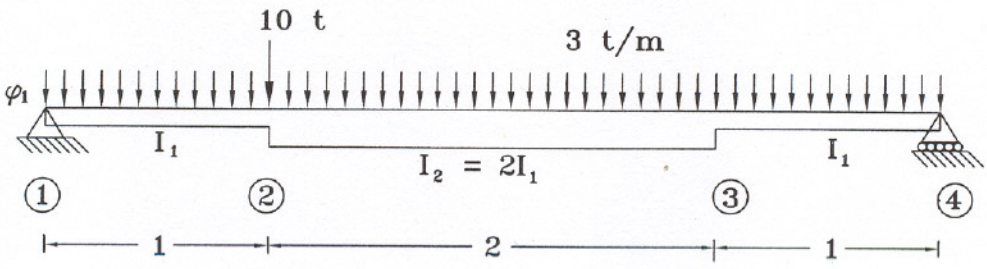


(ton)

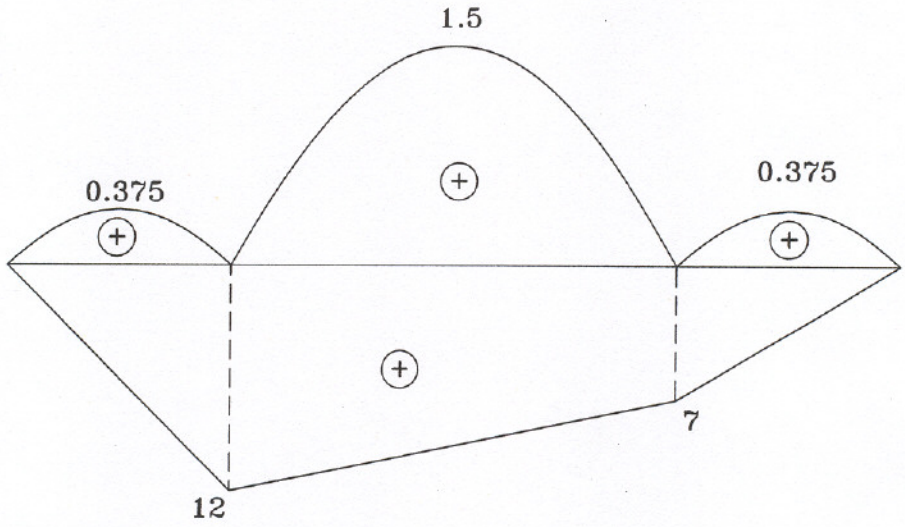
$$Ua = \frac{1}{2}\int \frac{N^2}{EA}\,dx \qquad Ua = \frac{11723.44}{EA}$$

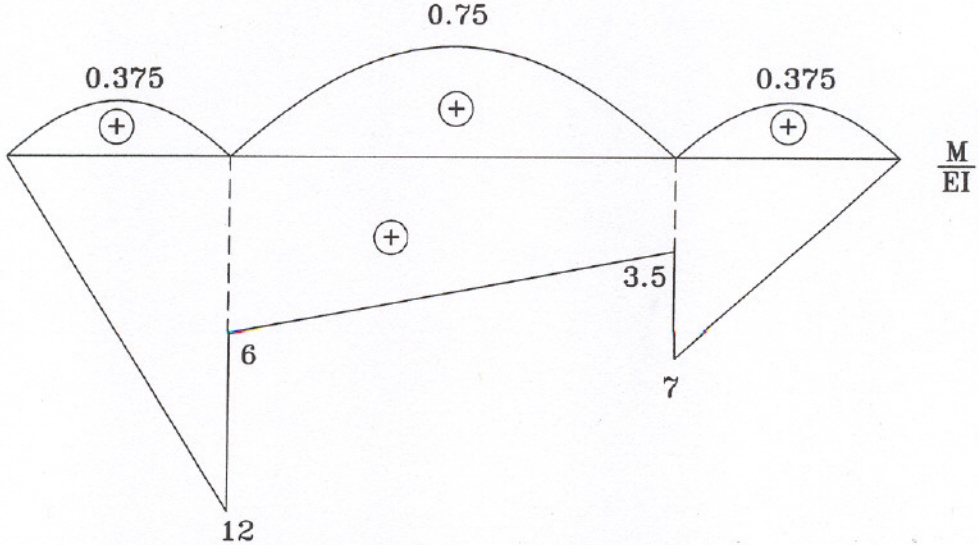
Problema 2.9

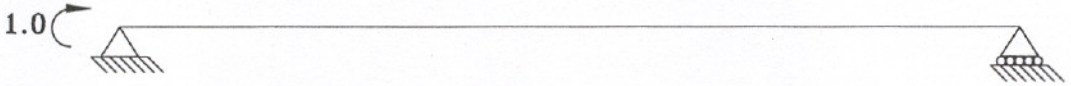
De la siguiente estructura, obtener el desplazamiento angular del punto ①, aplicando el método de trabajo virtual. Considere solo flexión.

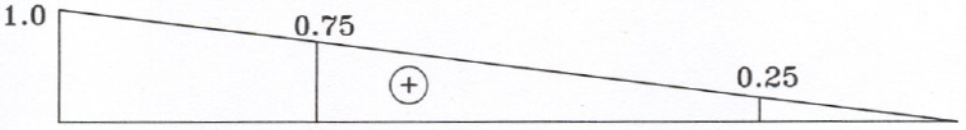


Resultados:







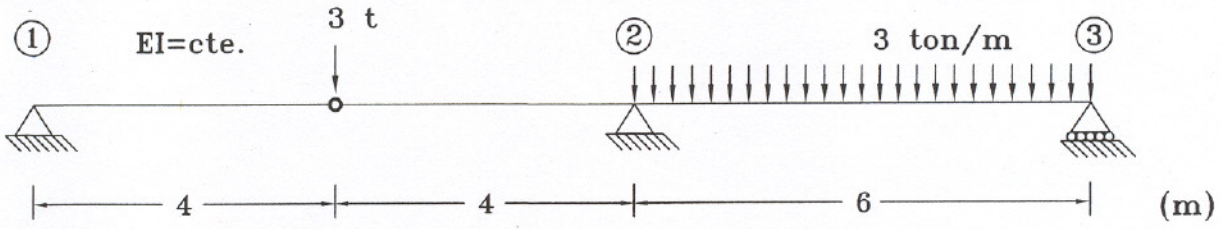


m
(ton−m)

$$\varphi_{①} = \int \frac{Mm}{EI}\, ds = \frac{11.79}{EI} \quad \left(\circlearrowright\right)$$

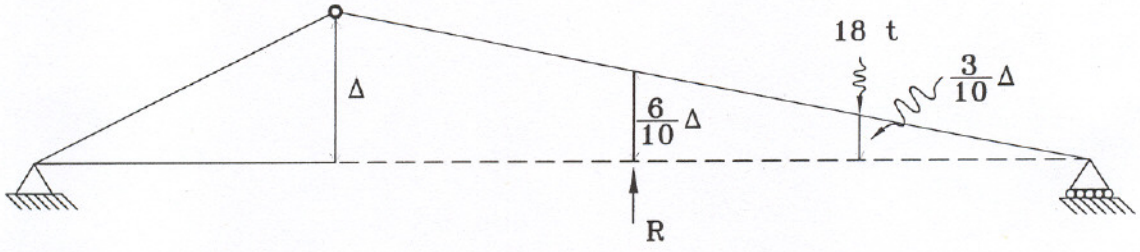
Problema 2.10

Encontrar reacciones aplicando el principio de desplazamiento virtual.



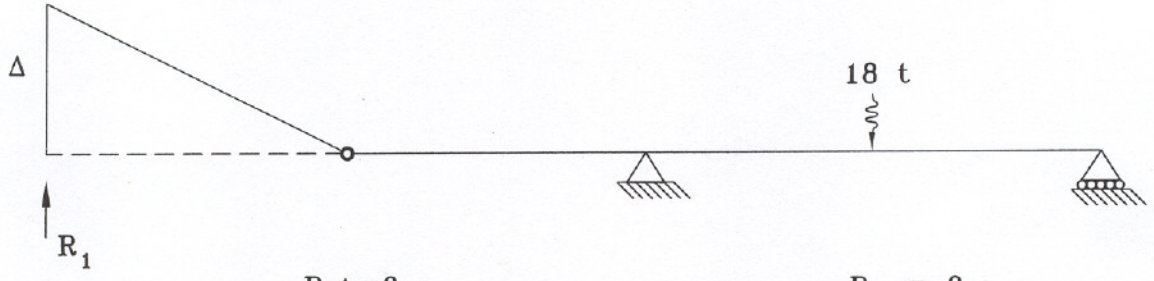
Resultados:

a)



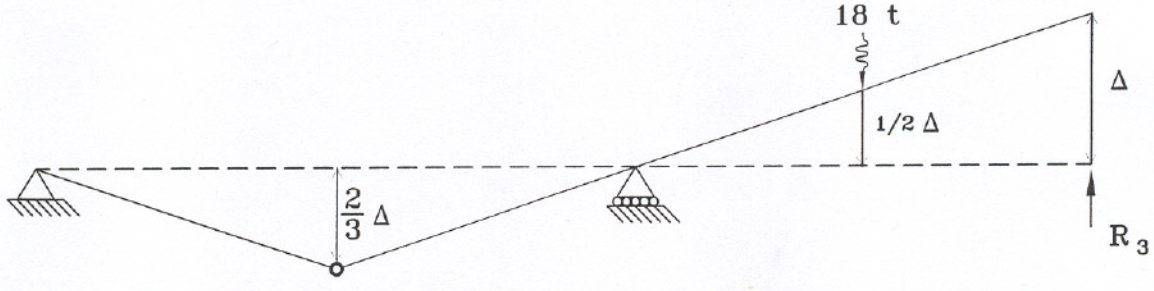
$$-3\Delta + R_2\left(\frac{6\Delta}{10}\right) - 18\left(\frac{3\Delta}{10}\right) = 0 \; ; \qquad R_2 = 14 \text{ t}$$

b)



$$R_1\Delta = 0 \; ; \qquad R_1 = 0$$

c)

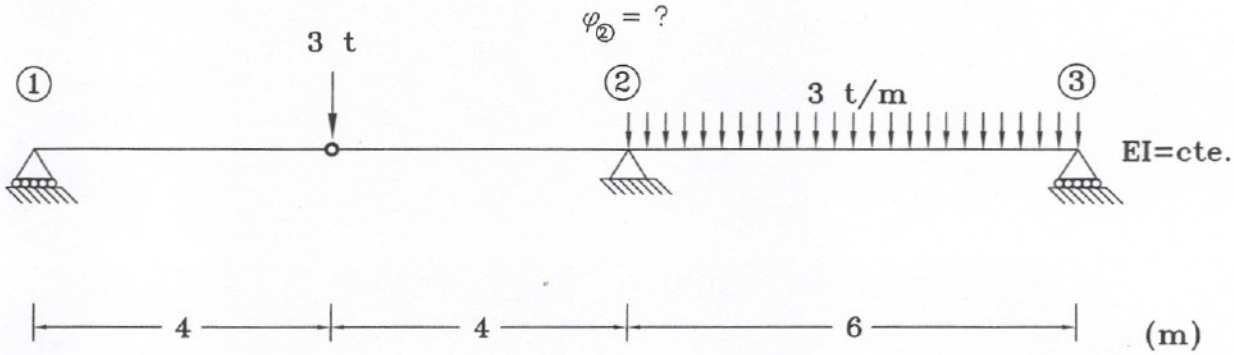


$$3\left(\frac{2\Delta}{3}\right) - 18\left(\frac{\Delta}{2}\right) + R_3\Delta = 0 \; ; \qquad R_3 = 7 \text{ t}$$

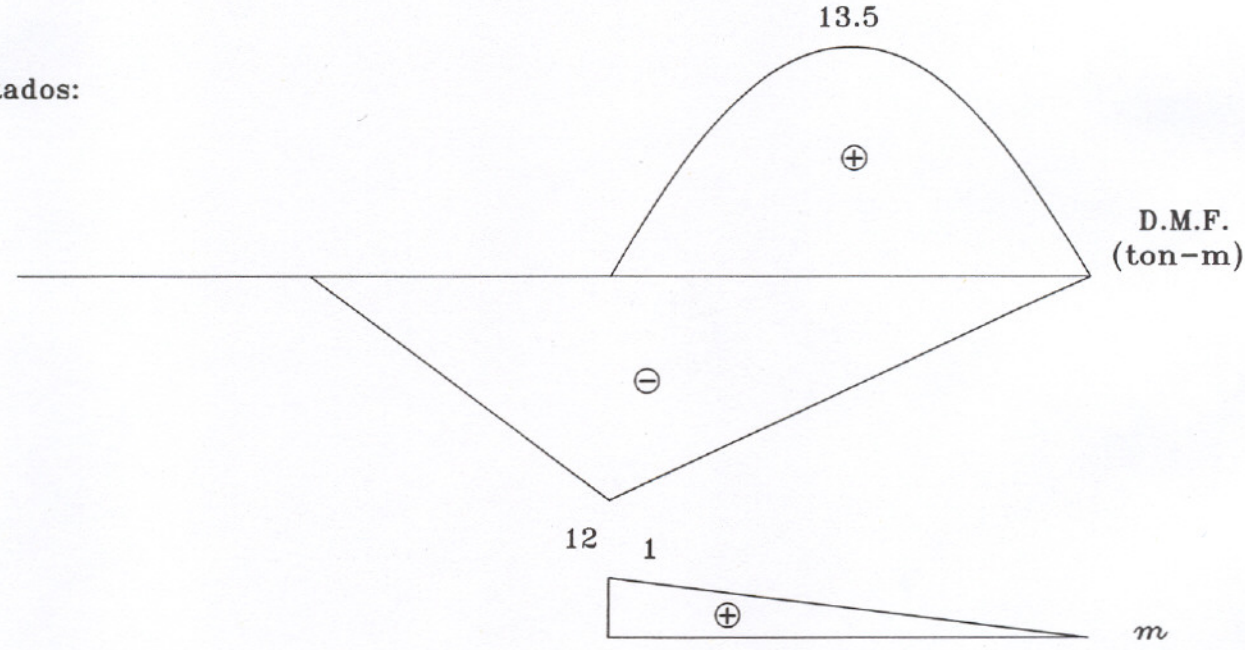
$R_1 = 0$ $\qquad$ $R_2 = 14$ t $\qquad$ $R_3 = 7$ t

Problema 2.11

Determine el giro del nudo ②



Resultados:

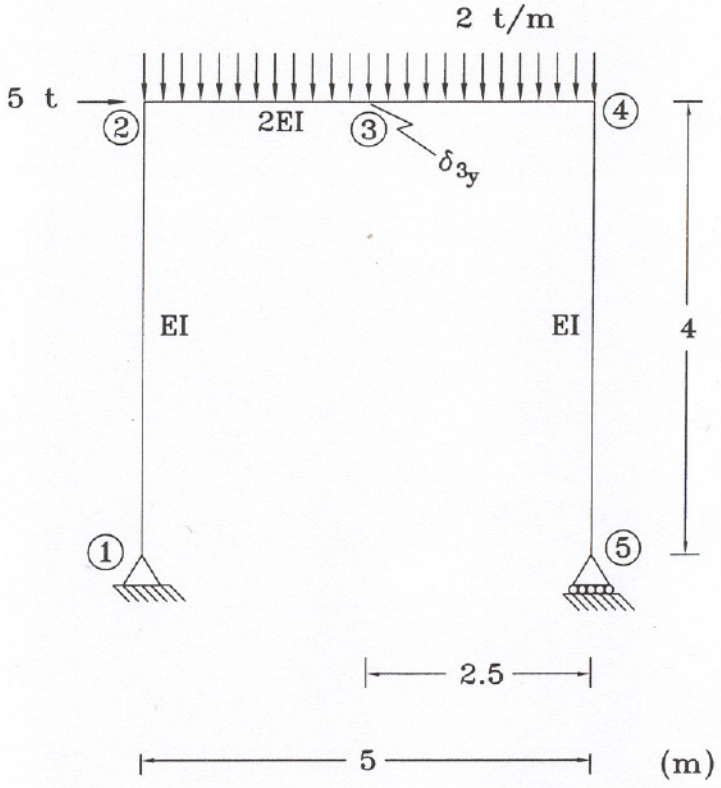


$$\varphi_{②} = \int \frac{Mm}{EI}\, ds$$

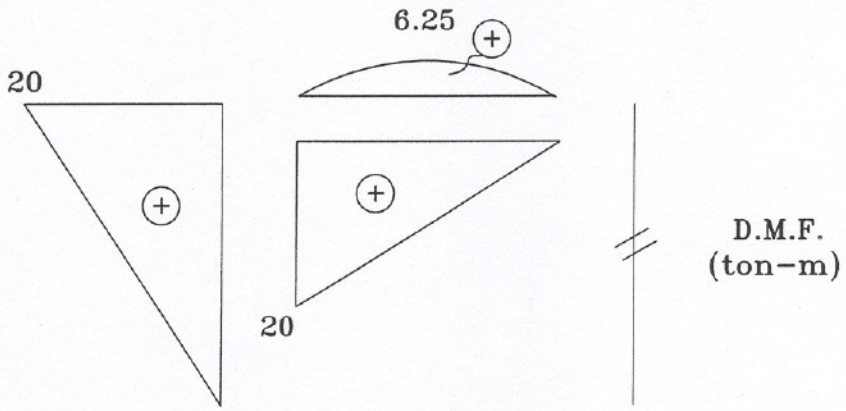
$$\varphi_{②} = \frac{3}{EI}$$

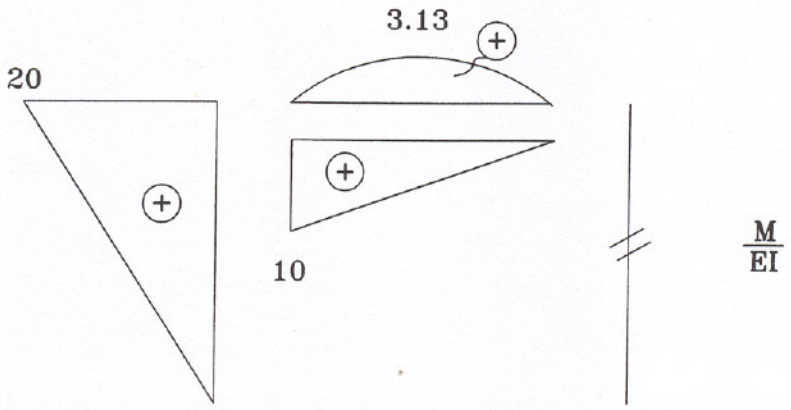
Problema 2.12

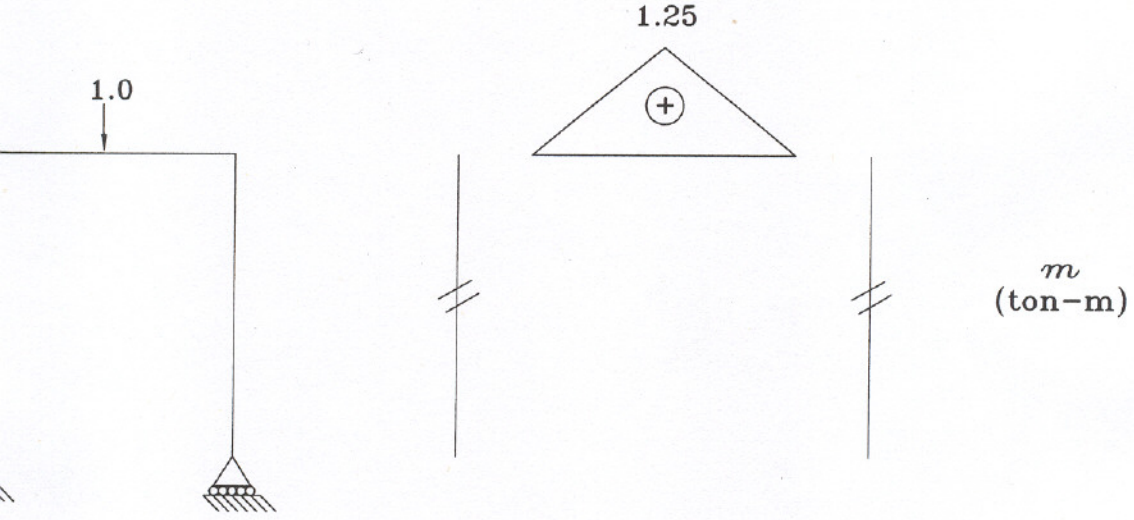
De la siguiente estructura, obtener el desplazamiento vertical del punto ③, aplicando el método de trabajo virtual. Considere solo flexión.



Resultados:



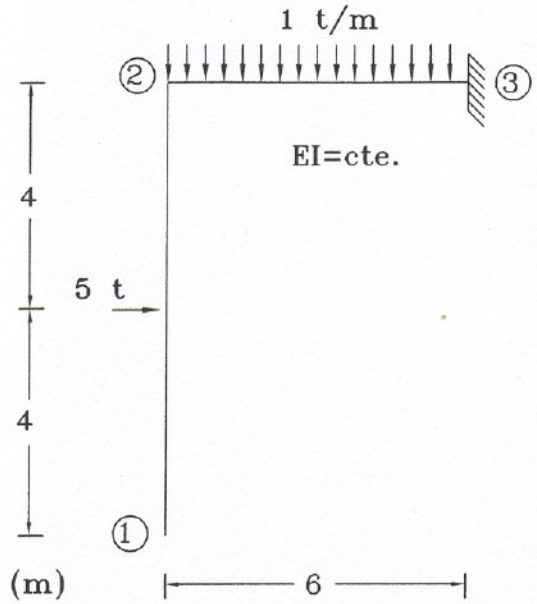




$$\delta_{3y} = \int \frac{Mm}{EI}\, ds = \frac{23.76}{EI} \quad (\downarrow)$$

Problema 2.13

De la siguiente estructura, obtener los desplazamientos que se indican, aplicando el método de trabajo virtual.

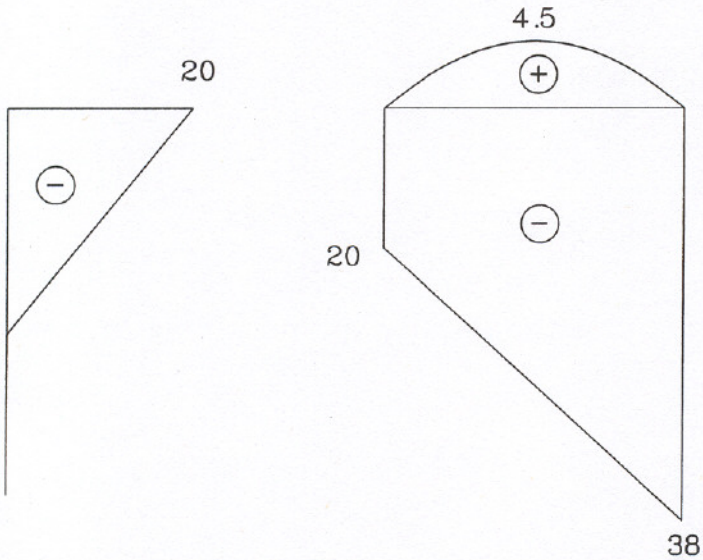


δ_{X1}=?

δ_{Y1}=?

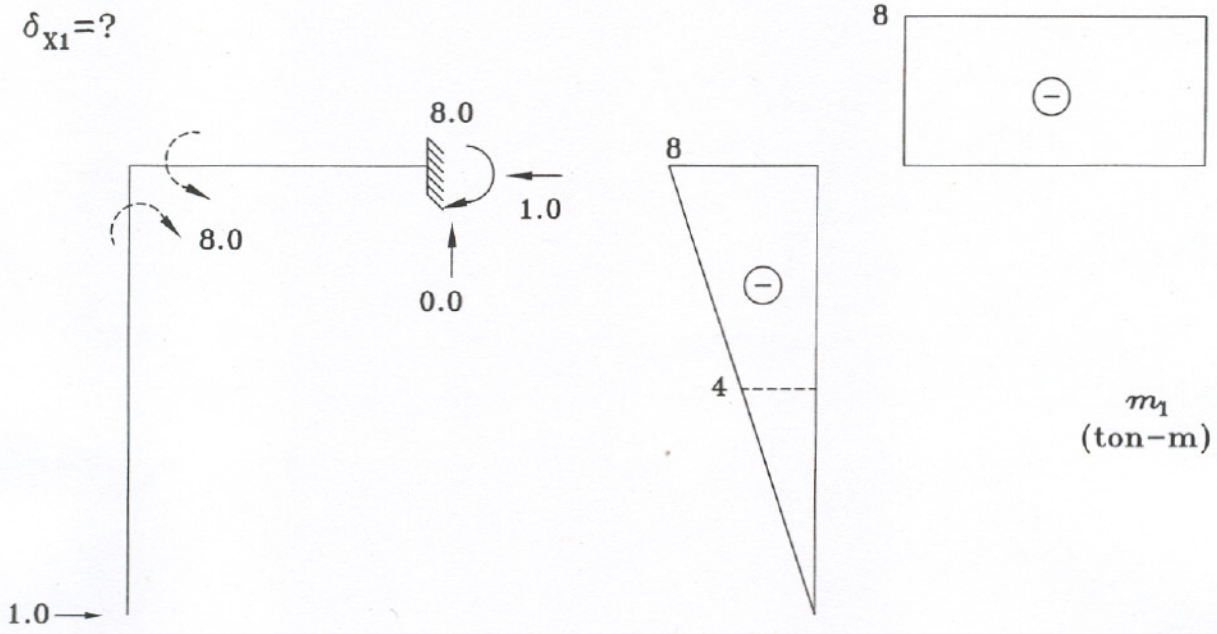
φ_1 =?

Resultados:



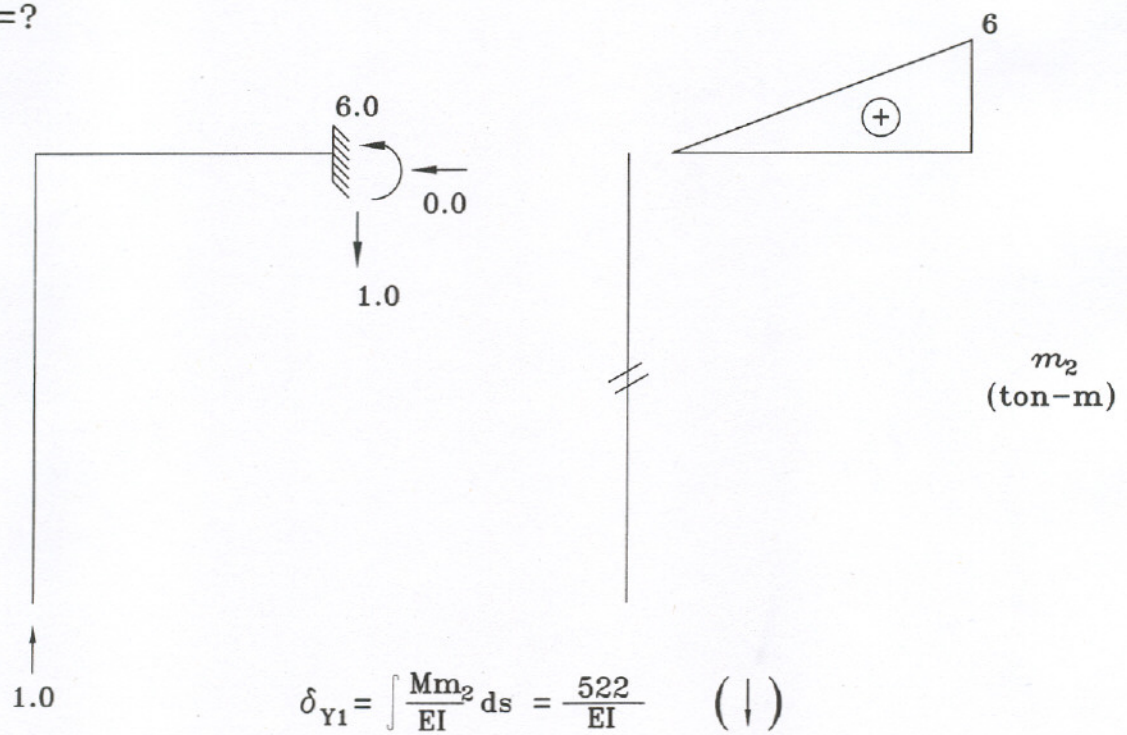
D.M.F.
(ton−m)

δ_{X1}=?



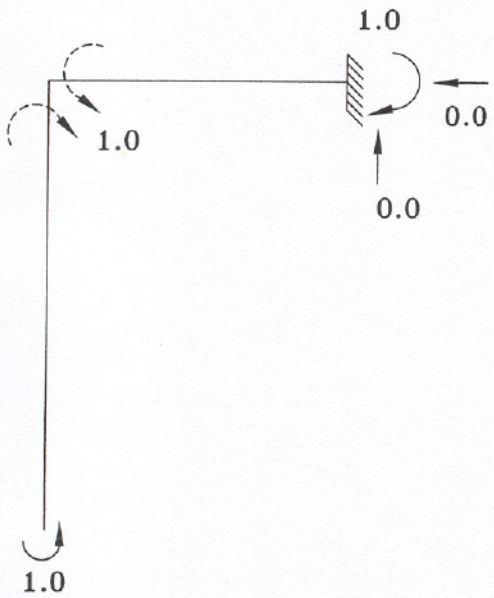
$$\delta_{X1} = \int \frac{Mm_1}{EI}\,ds = \frac{1514.67}{EI} \quad (\rightarrow)$$

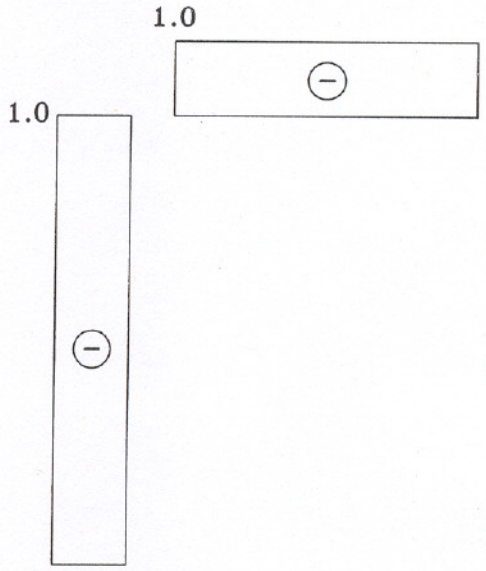
δ_{Y1}=?



$$\delta_{Y1} = \int \frac{Mm_2}{EI}\,ds = \frac{522}{EI} \quad (\downarrow)$$

$\varphi_1 = ?$



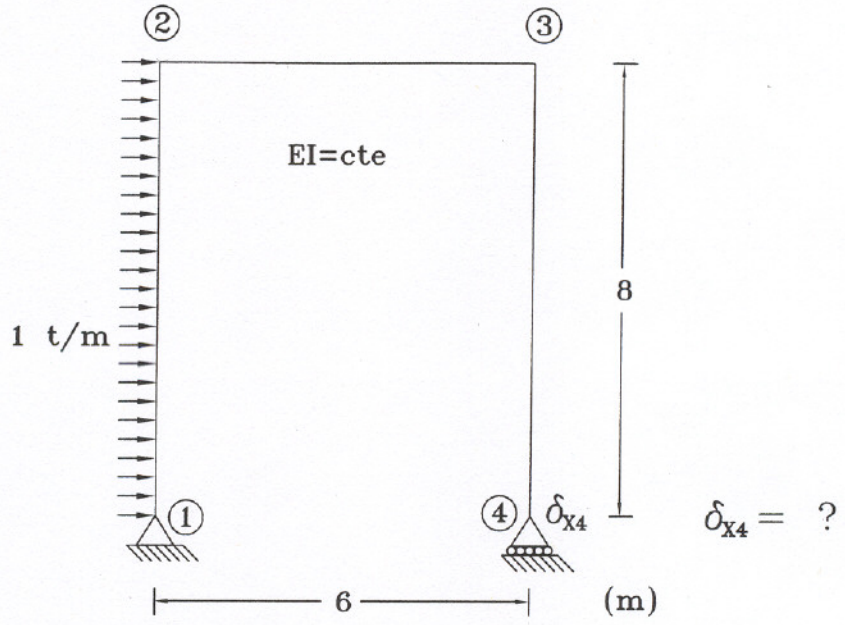


m_3
(ton−m)

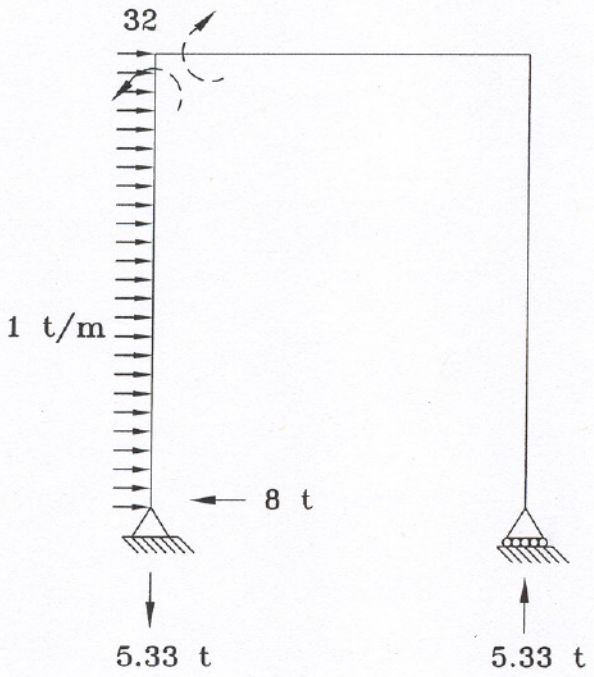
$$\varphi_1 = \int \frac{Mm_3}{EI}\, ds = \frac{196}{EI} \quad (\curvearrowleft)$$

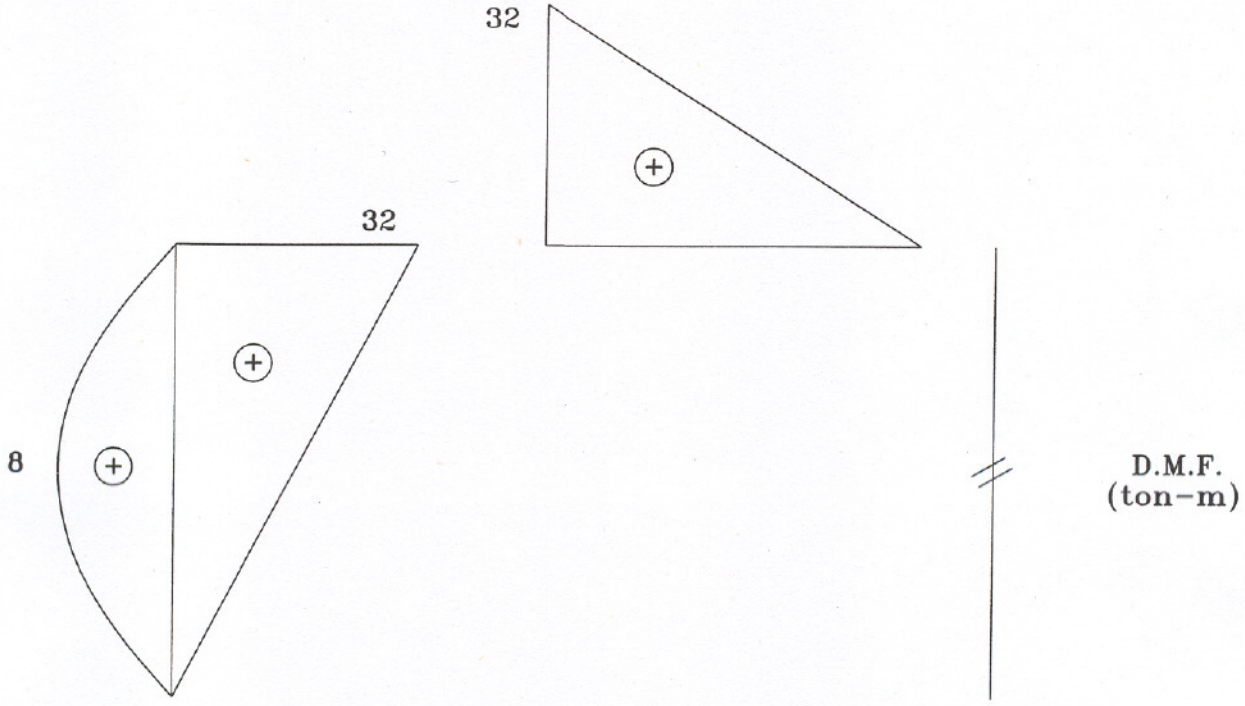
Problema 2.14

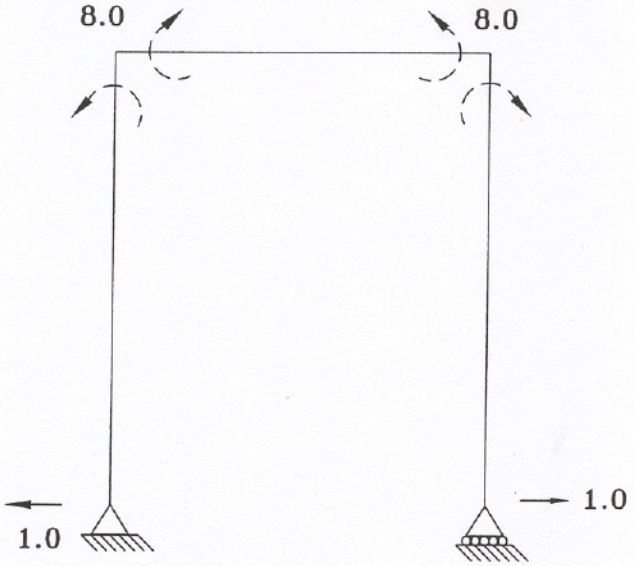
Calcular el desplazamiento que se indica. Considére solo el efecto de flexión

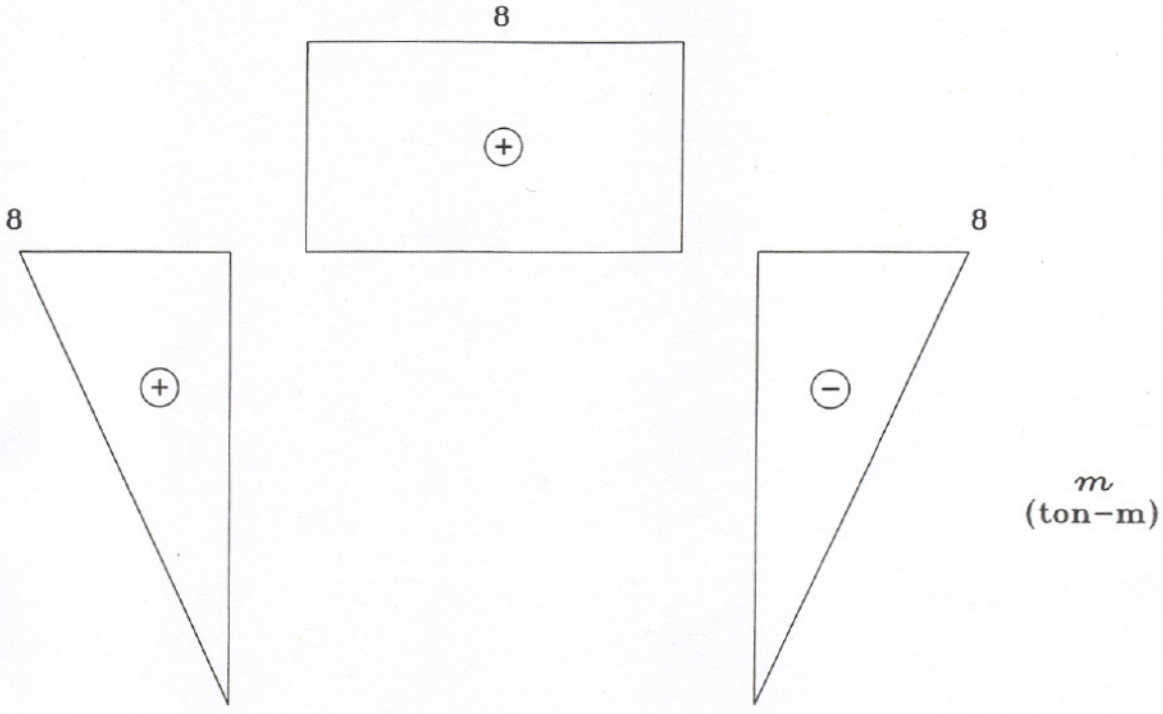


Resultados:




32
32
8
D.M.F.
(ton-m)

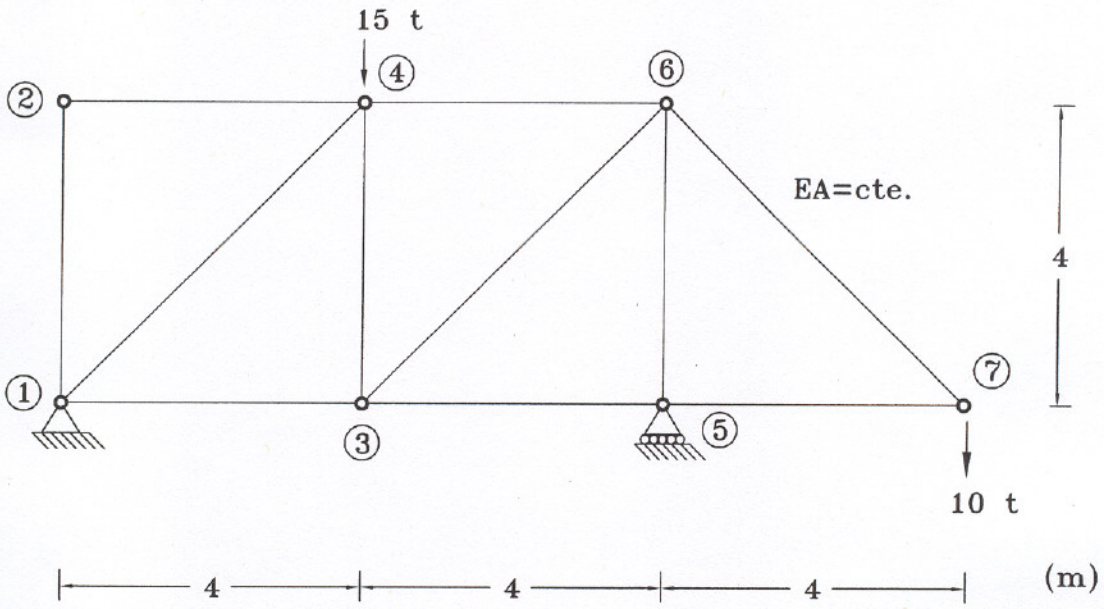

8.0
8.0
1.0
1.0
m
(ton-m)



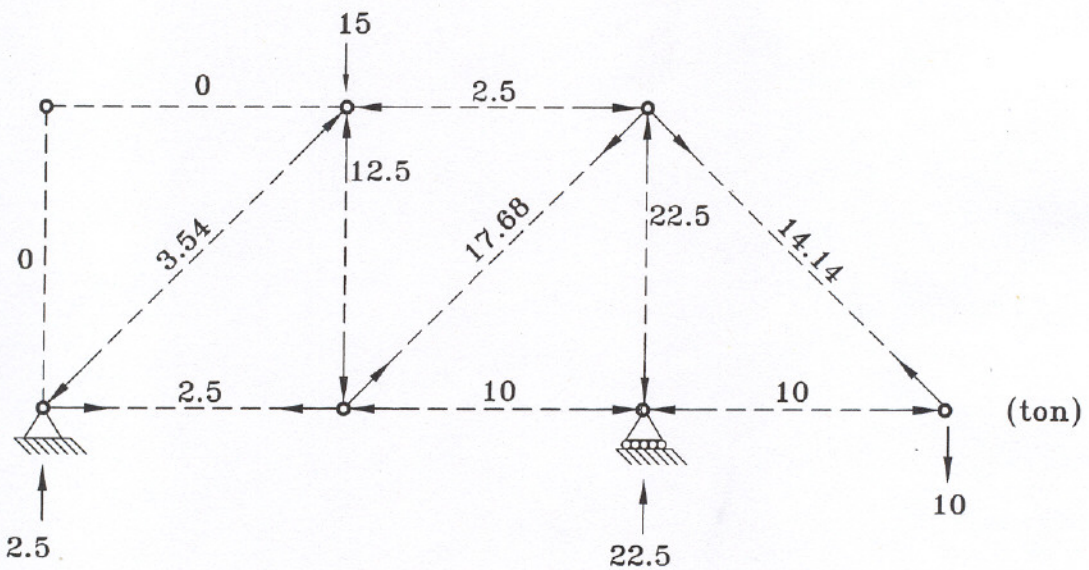
$$\delta_{X4} = \frac{1621.33}{EI} \quad (\rightarrow)$$

Problema 2.15

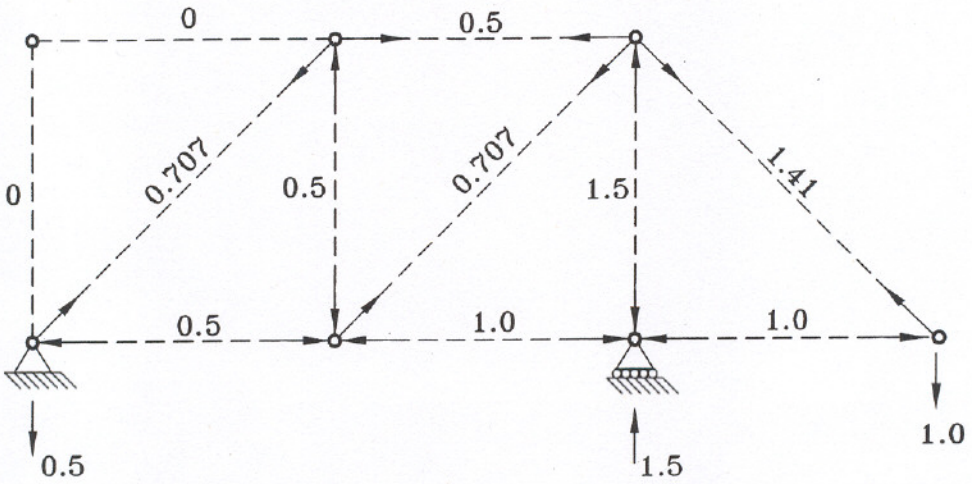
De la siguiente estructura, obtener los desplazamientos horizontal y vertical del nudo ⑦, aplicando el método de trabajo virtual.



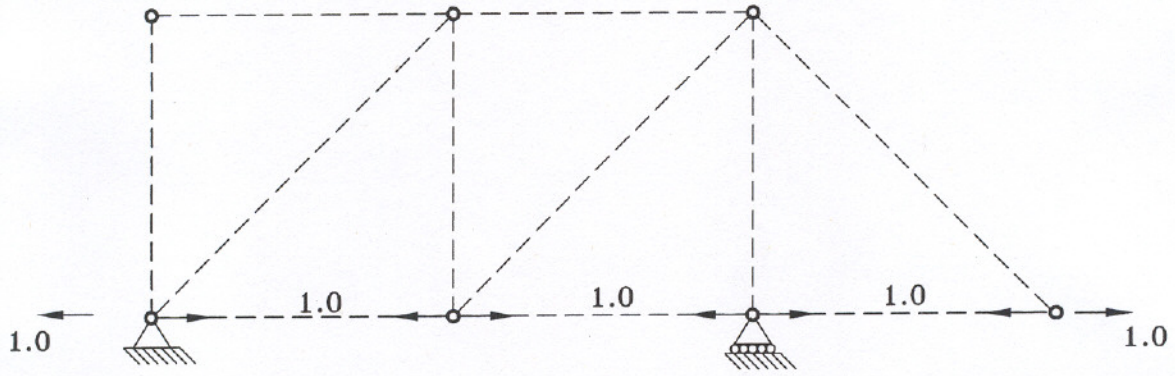
Resultados:



Para δ_{Y7} , n_1 (ton)



Para δ_{X7} , n_2 (ton)



$$\delta_{Y7} = \sum_{i=1}^{11} \frac{Nn_1 L}{EA} = \frac{87.5}{EA} \quad (\downarrow)$$

$$\delta_{X7} = \sum_{i=1}^{11} \frac{Nn_2 L}{EA} = \frac{17.5}{EA} \quad (\rightarrow)$$