ROCKET DYNAMICS

Drage equation $D = qAC_D$ | Delta V neglecting drag and gravity $\Delta v = I_{sp} g_0 \ln \frac{m_0}{m_f}$

$m_o = m_E + m_p + m_{PL}$
$m_f = m_E + m_{PL}$
m_E ≡ Empty / structural mass
m_p ≡ Propellant mass
m_{PL} ≡ Payload mass
λ ≡ Payload ratio
ε ≡ Structural ratio
D, C_D ≡ Drag, Drag coefficient

$q = ½\rho v^2$ ≡ Dynamic pressure
A ≡ Frontal area of the vehicle
c ≡ Effective exhaust velocity
c_a ≡ Exhaust speed relative to the rocket
p_a ≡ Static pressure of the atmosphere
p_e ≡ Rocket nozzle pressure
$\dot m_e$ ≡ Rate at which exhaust mass flows across the nozzle exit plane

A_e ≡ Nozzle exit area
T ≡ Thrust
I_{sp} ≡ Specific impulse
n ≡ Mass ratio
Δt ≡ Burn time
$\Delta v_D, \Delta v_G$ ≡ Drag loss, gravity loss
γ ≡ Flight path angle
v_{bo} ≡ Burnout speed

Specific Impuls $I_{sp} = \frac{T}{\dot m_e g_o}$ | Effective exaust velocity $c = c_a + \frac{(p_e - p_a)A_e}{\dot m_e}$ | Thrust $T = \dot m_e c$

Mass Ratio $n = \frac{m_o}{m_f}$ | Burn Time $\Delta t = \frac{n-1}{n}\frac{I_{sp}}{T/m_o g_o}$ | Payload Ratio $\lambda = \frac{m_{PL}}{m_E + m_p} = \frac{m_{PL}}{m_o - m_{PL}}$

Structural Ratio $\varepsilon = \frac{m_E}{m_E + m_p} = \frac{m_E}{m_o - m_{PL}}$ | Delta V $\Delta v = I_{sp} g_o \ln \frac{m_o}{m_f} - \Delta v_D - \Delta v_G$

Burnout Speed $v_{bo} = I_{sp} g_o \ln n = I_{sp} g_o \ln \frac{1+\lambda}{\varepsilon + \lambda}$ | Drag and gravity loss $\Delta v_D = \int_{t_o}^{t_f} \frac{D}{m} dt \quad \Delta v_G = \int_{t_o}^{t_f} g \sin\gamma \, dt$

Optimization procedure to an N- stage vehicle
η – Langrange multiplier

$$h = \sum_{i=1}^{N}[\ln(1-\varepsilon_i) + \ln n_i - \ln(1-\varepsilon_i n_i)] - \eta\left(v_{bo} - \sum_{i=1}^{N} c_i \ln n_i\right)$$

IDEAL GAS RELATIONS

Ideal gas model $pv = RT \quad u = u(T) \quad h = h(T) = u(T) + RT$

Change in specific internal energy $u(T_2) - u(T_1) = \int_{T_1}^{T_2} c_v(T)\, dT$ | For constant C_v $u(T_2) - u(T_1) = c_v(T_2 - T_1)$

Change in specific enthalpy $h(T_2) - h(T_1) = \int_{T_1}^{T_2} c_p(T)\, dT$ | For constant C_p $h(T_2) - h(T_1) = c_p(T_2 - T_1)$

COMPRESSIBLE FLOW IN NOZZLES AND DIFFUSERS

Momentum equation for steady-state one dimension flow $\mathbf{F} = \dot m(\mathrm{V}_2 - \mathrm{V}_1)$

Ideal gas velocity of sound $c = \sqrt{kRT}$

Mach number $M = \mathrm{V}/c$

Stagnation enthalpy $h_o = h + \mathrm{V}^2/2$

Isentropic flow funtion relating temperature and stagnation temperature (constant k) $\frac{T_o}{T} = 1 + \frac{k-1}{2}M^2$

Isentropic flow funtion relating pressure and stagnation pressure (constant k) $\frac{p_o}{p} = \left(\frac{T_o}{T}\right)^{k/(k-1)} = \left(1 + \frac{k-1}{2}M^2\right)^{k/(k-1)}$

VALISPACE

EMPOWERING ENGINEERS

SPACE CHEAT SHEET

contact-us@valispace.com
www.valispace.com

CONSTANTS

Frequent Constants

geostationary orbit altitude	h_{GEO}	35786 km
Low Earth Orbit altitude	h_{LEO}	120-3000 km
speed of light	c	299792.458 km/s
universal gas constant (ideal gas)	R	8.3144598 J mol^{-1} K^{-1}
Moon distance from Earth	d	384402 km
Moon radius	R	1737 km
Moon surface acceleration	g	1.625 m/s^2
Mars radius	R	3396 km
Mars surface acceleration	g	3.72 m/s^2
Newtonian constant of gravitation	G	6.673 848 × 10^{-11} m^3 kg^{-1} s^{-2}
standard acceleration of gravity	g_0	9.806 65 m s^{-2}
Earth mass	M	5.974 × 10^{24} kg
Earth radius (volumetric mean)	R	6.378 × 10^6 m
solar luminosity	L	3.846 × 10^{26} W
solar mass	M	1.989 × 10^{30} kg
solar radius (volumetric mean)	R	6.960 × 10^8 m
Jupiter mass	M	1.899 × 10^{27} kg
Jupiter radius	R	6.991 × 10^7 m
astronomical unit	1 UA	1.495 979 × 10^{11} m
parsec pc	1 pc	3.085 678 × 10^{16} m
Sidereal year	1 sidereal year	365.26 days
Sidereal day	1 sidereal day	86164.09 s

Regions of the Electromagnetic Spectrum

	Wavelenth (m)	Frequency (Hz)	Photon Energy (J)
Radio	> 1 × 10^{-1}	< 3 × 10^9	< 2 × 10^{-24}
Microwave	1 × 10^{-3} - 1 × 10^{-1}	3 × 10^9 - 3 × 10^{11}	2 × 10^{-24} - 2 × 10^{-22}
Infrared	7 × 10^{-7} - 1 × 10^{-3}	3 × 10^{11} - 4 × 10^{14}	2 × 10^{-22} - 3 × 10^{-19}
Optical	4 × 10^{-7} - 7 × 10^{-7}	4 × 10^{14} - 7.5 × 10^{14}	3 × 10^{-19} - 5 × 10^{-19}
UV	1 × 10^{-8} - 4 × 10^{-7}	7.5 × 10^{14} - 3 × 10^{16}	5 × 10^{-19} - 2 × 10^{-17}
X-ray	1 × 10^{-11} - 1 × 10^{-8}	3 × 10^{16} - 3 × 10^{19}	2 × 10^{-17} - 2 × 10^{-14}
Gamma-ray	< 1 × 10^{-11}	> 3 × 10^{19}	2 × 10^{-14}

Astronomical Data

Object	Radius (km)	Mass (kg)	Sidereal Rotation Period	Inclination of Equator to Orbit Plane	Semimajor Axis of Orbit (km)	Orbit Eccentricity	Inclination of Orbit to the Ecliptic Plane	Orbit Sidereal Period
Sun	696000	1.989 × 10^{30}	25.38d	7.25°	–	–	—	—
Mercury	2440	330.2 × 10^{21}	58.65d	0.01°	57.91 × 10^6	0.2056	7.00°	87.97d
Venus	6052	4.869 × 10^{24}	243d*	177.4°	108.2 × 10^6	0.0067	3.39°	224.7d
Earth	6378	5.974 × 10^{24}	23.9345h	23.45°	149.6 × 10^6	0.0167	0.00°	365.256d
(Moon)	1737	73.48 × 10^{21}	27.32d	6.68°	384.4 × 10^3	0.0549	5.145°	27.322d
Mars	3396	641.9 × 10^{21}	24.62h	25.19°	227.9 × 10^6	0.0935	1.850°	1.881y
Jupiter	71490	1.899 × 10^{27}	9.925h	3.13°	778.6 × 10^6	0.0489	1.304°	11.86y
Saturn	60270	568.5 × 10^{24}	10.66h	26.73°	1.433 × 10^9	0.0565	2.485°	29.46y
Uranus	25560	86.83 × 10^{24}	17.24h*	97.77°	2.872 × 10^9	0.0457	0.772°	84.01y
Neptune	24760	102.4 × 10^{24}	16.11h	28.32°	4.495 × 10^9	0.0113	1.769°	164.8y
(Pluto)	1195	12.5 × 10^{21}	6.387d	122.5°	5.870 × 10^9	0.2444	17.16°	247.7y

*Retrograde

Gravitational parameters

Body	Body Symbol	μ=GM (km^3/s^2)	SOI radius (km)
Sun	☉	1.32712440017987 × 10^{11}	—
Mercury	☿	2.2032080486 × 10^4	112 000
Venus	♀	3.24858598826 × 10^5	616 000
Earth	⊕	3.98600435608 × 10^5	925 000
(Moon)	☽	4.902799108 × 10^3	66 100
Mars*	♂	4.2828314258 × 10^4	577 000
Jupiter*	♃	1.2671276785779 6×10^8	48 200 000
Saturn*	♄	3.7940626061137 × 10^7	54 800 000
Uranus*	⛢	5.794549007072 × 10^6	51 800 000
Neptune*	♆	6.836534063879 × 10^6	86 600 000
(Pluto)*	♇	9.81600888 × 10^2	3 080 000

*Includes the gravitational parameters of the planet's satellites

RELATIVE MOTION

$i \equiv$ inertial frame, $s \equiv$ reference frame

$\omega^{si} \equiv$ angular velocity of the s frame with respect to the i frame

$r \equiv$ position vector in s frame, $v \equiv$ velocity, $a \equiv$ acceleration

$R \equiv$ position vector of the origin of s–frame

$$^{i}v = \frac{^{i}d}{dt}r = \frac{^{s}d}{dt}r + \omega^{si} \times r$$

$$\frac{^{i}d^2}{dt^2}(R + r) = \frac{^{i}d^2}{dt^2}R + \frac{^{s}d^2}{dt^2}r + 2\omega^{si} \times \frac{^{s}d}{dt}r + \left(\frac{^{s}d}{dt}\omega^{si}\right) \times r + \omega^{si} \times (\omega^{si} \times r)$$

Polar Coordinates

Unit Vectors

$$\boldsymbol{e}_r = \cos\theta \boldsymbol{i} + \sin\theta \boldsymbol{j}$$
$$\boldsymbol{e}_\theta = -\sin\theta \boldsymbol{i} + \cos\theta \boldsymbol{j}$$

Kinematic Equations

$$\boldsymbol{r} = r\boldsymbol{e} \qquad \boldsymbol{v} = \dot{r}\boldsymbol{e}_r + r\dot{\theta}\boldsymbol{e}_\theta$$
$$\boldsymbol{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\boldsymbol{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\boldsymbol{e}_\theta$$

Motion Equations

$$F_r = m\,a_r = m\left(\ddot{r} - r\dot{\theta}^2\right)$$
$$F_\theta = m\,a_\theta = m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)$$

Cylindrical Coordinates

Unit Vectors

$$\boldsymbol{e}_r = \cos\theta \boldsymbol{i} + \sin\theta \boldsymbol{j}$$
$$\boldsymbol{e}_\theta = -\sin\theta \boldsymbol{i} + \cos\theta \boldsymbol{j}$$

Kinematic Equations

$$\boldsymbol{r} = r\,\boldsymbol{e}_r + z\,\boldsymbol{k} \qquad \boldsymbol{v} = \dot{r}\,\boldsymbol{e}_r + r\dot{\theta}\,\boldsymbol{e}_\theta + \dot{z}\,\boldsymbol{k}$$
$$\boldsymbol{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\boldsymbol{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\boldsymbol{e}_\theta + \ddot{z}\boldsymbol{k}$$

Motion Equations

$$F_r = m\,a_r = m\left(\ddot{r} - r\dot{\theta}^2\right)$$
$$F_\theta = m\,a_\theta = m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)$$
$$F_z = m\,a_z = m\,\ddot{z}$$

Spherical Coordinates

Unit Vectors

$$\boldsymbol{e}_r = \cos\theta\,\cos\phi\,\boldsymbol{i} + \sin\theta\,\cos\phi\,\boldsymbol{j} + \sin\phi\,\boldsymbol{k}$$
$$\boldsymbol{e}_\theta = -\sin\theta\,\boldsymbol{i} + \cos\theta\,\boldsymbol{j}$$
$$\boldsymbol{e}_\phi = -\cos\theta\,\sin\phi\,\boldsymbol{i} - \sin\theta\,\sin\phi\,\boldsymbol{j} + \cos\phi\,\boldsymbol{k}$$

Kinematic Equations

$$\boldsymbol{r} = r\,\boldsymbol{e}_r$$
$$\boldsymbol{v} = \dot{r}\boldsymbol{e}_r + r\dot{\theta}\cos\phi\,\boldsymbol{e}_\theta + r\dot{\phi}\,\boldsymbol{e}_\phi$$
$$\boldsymbol{a} = \left(\ddot{r} - r\dot{\theta}^2\cos^2\phi - r\dot{\phi}^2\right)\boldsymbol{e}_r + \left(2\dot{r}\dot{\theta}\cos\phi + r\ddot{\theta}\cos\phi - 2r\dot{\theta}\dot{\phi}\sin\phi\right)\boldsymbol{e}_\theta + \left(2\dot{r}\dot{\phi} + r\dot{\theta}^2\sin\phi\,\cos\phi + r\ddot{\phi}\right)\boldsymbol{e}_\phi$$

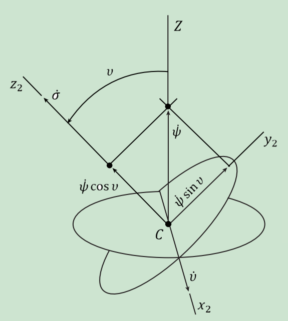
Equations of Motion

$$F_r = m\,a_r = m\left(\ddot{r} - r\dot{\theta}^2\cos^2\phi - r\dot{\phi}^2\right)$$
$$F_\theta = m\,a_\theta = m\left(2\dot{r}\dot{\theta}\cos\phi + r\ddot{\theta}\cos\phi - 2r\dot{\theta}\dot{\phi}\sin\phi\right)$$
$$F_\phi = m\,a_\phi = m\left(2\dot{r}\dot{\phi} + r\dot{\theta}^2\sin\phi\cos\phi + r\ddot{\phi}\right)$$

SATELLITE ATTITUDE DYNAMICS

Dual-Spin Spacecraft - Axisymmetric Body

δ – spin, ψ – precession, ν – nutation



Euler's equations for torque-free motion

$$\begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} = \begin{bmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} A\omega_x \\ B\omega_y \\ C\omega_z \end{bmatrix}$$

Angular Position of the body in the spinning reference frame

$$\left\{\vec{\Omega}\right\}_{x_2y_2z_2 \equiv xyz} = \vec{\dot{\psi}} + \vec{\dot{\nu}} = \begin{bmatrix} \dot{\nu} \\ \dot{\psi}\sin\nu \\ \dot{\psi}\cos\nu \end{bmatrix}$$

Angular velocity in the spinning reference frame

$$\{\vec{\omega}\}_{x_2y_2z_2 \equiv xyz} = \left\{\vec{\Omega}\right\}_{x_2y_2z_2 \equiv xyz} + \vec{\dot{\sigma}} = \begin{bmatrix} \dot{\nu} \\ \dot{\psi}\sin\nu \\ \dot{\psi}\cos\nu + \dot{\sigma} \end{bmatrix}$$

Angular moment components

$$(\vec{H}_c)_i = \begin{bmatrix} A\dot{\nu} \\ A\dot{\psi}\sin\nu \\ C(\dot{\psi}\cos\nu + \dot{\sigma}) \end{bmatrix}$$

γ – wobble angle

Relations between precession and spin

$$\dot{\psi} = \frac{C}{(A - C)\cos\nu}\dot{\sigma} \qquad \tan\gamma = \frac{\omega_y}{\omega_z} = \frac{\dot{\psi}\sin\nu}{\dot{\psi}\cos\nu + \dot{\sigma}} \qquad \tan\gamma = \frac{C}{A}\tan\nu$$

ORBITS

VALISPACE

Central Force

$H \equiv$ Angular momentum

$h \equiv$ specific angular momentum

$\frac{dA}{dt} \equiv$ areal velocity

$\mu \equiv$ gravitational parameter

$v \equiv$ velocity

$E \equiv$ Total Energy

$U \equiv$ Potential Energy

$r \equiv$ position

Formal solution of the central force: $$t - t_0 = \int_{r_0}^{r} \frac{\mathrm{d}r}{\pm\sqrt{\frac{2}{m}\left(E - U(r)\right) - \frac{h^2}{r^2}}}$$

Kepler's 2nd law: $$\frac{dA}{dt} = \frac{H_0}{2m} = \frac{h}{2} = constant$$

Conservation of angular momentum: $$\vec{H}_0 = \vec{r} \times m\vec{v} = \text{Cte}$$

Specific angular momentum: $$h = r^2\dot{\theta}$$

Central force: $$\vec{F} = F\frac{\vec{r}}{r}$$

Gravitational central force: $$F = -\frac{GMm}{r^2}e_r;\ f = -\frac{\mu}{r^2}$$

Ellipse Properties

$e \equiv$ eccentricity

$a \equiv$ semimajor axis

$b \equiv$ semiminor axis

$p \equiv$ semi lactus rectum / parameter

$r_p \equiv$ periapsis

$r_a \equiv$ apoapsis

$F \equiv$ Focus , $C \equiv$ Center

$c \equiv$ linear eccencitry

$\varepsilon \equiv$ specific energy

$v_r \equiv$ radial velocity

$\gamma \equiv$ flight path angle

$T \equiv$ orbital period

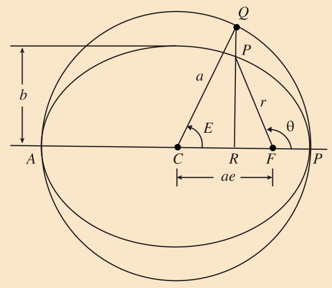
$n \equiv$ mean motion

$M \equiv$ mean anomaly

$E \equiv$ eccentric anomaly

θ ≡ true anomaly

$v_{esc} \equiv$ escape velocity



$$2a = r_p + r_a \qquad b = a\sqrt{1 - e^2} \qquad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$r_p = a(1 - e) = \frac{p}{1 + e} \qquad \bar{C}F = c = ae \qquad Area = ab\pi$$

$$r_a = a(1 + e) \qquad e = \sqrt{1 - \left(\frac{b}{a}\right)^2} \qquad p = a(1 - e^2) = \frac{h^2}{\mu}$$

Elliptical Orbits

Gravitational Parameter: $$\mu = GM$$

Specific energy: $$\varepsilon = -\frac{\mu}{2a}$$

eccentricity: $$e = \sqrt{1 + 2\varepsilon\left(\frac{h}{\mu}\right)^2}$$

Mean motion: $$n = \frac{2\pi}{T} = \sqrt{\frac{\mu}{a^3}}$$

Radial velocity: $$v_r = \frac{\mu}{h}e\sin\theta$$

Escape velocity: $$v_{esc} = \sqrt{\frac{2\mu}{r}}$$

Orbital period: $$T = \frac{2\pi}{\sqrt{\mu}}\,a^{3/2}$$

Kepler's 3rd law: $$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{a_2}{a_1}\right)^3$$

Eccentricity vector: $$\vec{e} = \frac{1}{\mu}\left(\vec{v} \times \vec{h} - \frac{\mu\vec{r}}{r}\right)$$

Orbit equation Kepler's 1st law: $$r = \frac{h^2}{\mu}\frac{1}{1 + e\cos\theta} = \frac{p}{1 + e\cos\theta}$$

vis - viva equation: $$\frac{v^2}{2} - \frac{\mu}{r} = \varepsilon = cte;\ v = \sqrt{2\mu\left(\frac{1}{r} - \frac{1}{2a}\right)}$$

Kepler's Equation: $$M = n(t - t_0) = E - e\sin E$$

E as function of θ: $$\tan E = \frac{\sqrt{1 - e^2}\sin\theta}{e + \cos\theta},\ \tan\frac{E}{2} = \sqrt{\frac{1 - e}{1 + e}}\tan\frac{\theta}{2}$$

Circular Orbit

Orbit equation: $r = \frac{h^2}{\mu}$ — velocity: $v = \sqrt{\frac{\mu}{r}}$

Time versus true anomaly: $t = \frac{\theta}{2\pi}T$ — specific energy: $\varepsilon = -\frac{\mu}{2r}$

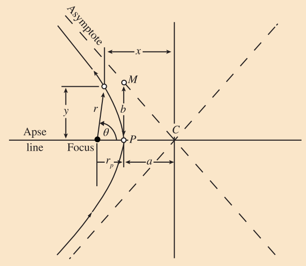
Orbital period: $T = \frac{2\pi}{\sqrt{\mu}}r^{3/2}$

Parabolic orbits

Parabolic mean anomaly: $$M_p = \frac{\mu^2}{h^3}t$$

Barker's equation: $$2\sqrt{\frac{\mu}{p^3}}(t - t_0) = \tan\frac{\theta}{2} + \frac{1}{3}\tan^3\frac{\theta}{2}$$

Hyperbolic orbits



$$\sinh F = \frac{y}{b}$$

Hyperbolic mean anomaly: $$M_h = \frac{\mu^2}{h^3}(e^2 - 1)^{3/2}t$$

Kepler's equation for the hyperbola: $$\sqrt{\frac{\mu}{-a^3}}(t - t_0) = e\sinh F - F$$

True anomaly in terms of F: $$\cos\theta = \frac{e - \cosh F}{e\cosh F - 1}$$

Orbital Elements

$h \equiv$ specific angular momentum

$i \equiv$ inclination

$\Omega \equiv$ right ascension (RA) of the ascending node

$e \equiv$ eccentricity

$\omega \equiv$ argument of perigee

θ ≡ true anomaly

$\{i, \Omega, \varpi, a, e, T_0\}$

$\vec{n}$ – *nodes line*

$$a = -\frac{\mu}{v_0^2 - \frac{\mu}{r_0}} \qquad \vec{e} = \frac{1}{\mu}\left[\vec{v}_0 \times (\vec{r}_0 \times \vec{v}_0) - \mu\frac{\vec{r}_0}{r_0}\right] \qquad \cos\theta_0 = \frac{\vec{e}\cdot\vec{r}_0}{er_0}$$

$$\vec{n} = \frac{\vec{e}_Z \times \vec{h}}{|\vec{e}_Z \times \vec{h}|} \qquad \vec{n} = \cos\Omega\,\vec{e}_X + \sin\Omega\,\vec{e}_Y \qquad \cos i = \frac{\vec{e}_Z\cdot\vec{h}}{|\vec{h}|}$$

$$\cos\varpi = \frac{\vec{n}\cdot\vec{e}}{|\vec{e}|} \qquad \tan\theta_0 = \frac{\left(\frac{r_0v_0^2}{\mu}\right)\sin\gamma_0\cos\gamma_0}{\left(\frac{r_0v_0^2}{\mu}\right)\cos^2\gamma_0 - 1} \qquad e^2 = \left(\frac{r_0v_0^2}{\mu} - 1\right)^2\cos^2\gamma_0 + \sin^2\gamma_0$$

Effects of the Earth's oblateness

$$\dot{\Omega} = -\frac{3nJ_2R_\oplus^2}{2a^2(1 - e^2)^2}\cos i \qquad \dot{\varpi} = \frac{3nJ_2R_\oplus^2}{2a^2(1 - e^2)^2}\left(2 - \frac{5}{2}\sin^2 i\right)$$

Oblateness and second zonal harmonics

Planet	Oblateness	J_2
Mercury	0.000	60×10^{-6}
Venus	0.000	4.458×10^{-6}
Earth	0.003353	1.08263×10^{-3}
Mars	0.00648	1.96045×10^{-3}
Jupiter	0.06487	14.736×10^{-3}
Saturn	0.09796	16.298×10^{-3}
Uranus	0.02293	3.34343×10^{-3}
Neptune	0.01708	3.411×10^{-3}
(Moon)	0.0012	202.7×10^{-6}

3 bodies

Jacobi integral

$$\frac{1}{2}v^2 - \frac{1}{2}\left(x^2 + y^2\right) - \frac{1 - \mu}{r_1} - \frac{\mu}{r_2} = C$$

Orbital Maneuvers

Hohmann Transfer

ϕ – Phase angle $\qquad 2a_H = r_1 + r_2$

$$\Delta v_1 = \sqrt{2\mu\left(\frac{1}{r_1} - \frac{1}{2a_H}\right)} - \sqrt{\frac{\mu}{r_1}} \qquad \Delta v_2 = \sqrt{\frac{\mu}{r_2}} - \sqrt{2\mu\left(\frac{1}{r_2} - \frac{1}{2a_H}\right)}$$

time during transfer – $t_t = \frac{T_H}{2} = \pi\sqrt{\frac{a_H^3}{\mu}}$

$$T_{syn} = \frac{T_1T_2}{|T_1 - T_2|} \qquad \phi = \phi_f + (n_2 - n_1)t \qquad t_{wait} = \frac{-2\phi_f \pm 2\pi N}{n_2 - n_1}$$